AN ALTERNATIVE APPROACH FOR TEACHING THE INTEREST METHOD AMORTIZATION OF BOND PREMIUMS AND DISCOUNTS

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INTRODUCTION

Discounts and premiums occur when the issuance price of a bond is more (premium) or less (discount) than its principal or face value. This result is due to a difference between the market or effective rate of interest and the coupon or face interest rate of the bond on the issuance date. If the market rate exceeds the coupon rate, the bond is issued at a discount, and if the market rate is less than the coupon rate, the bond is issued at a premium. When there is a discount or premium, accountants for the issuing organization journalize the transaction on the issue date as follows:

> Debit Cash
> Debit Bond Discount
> > Credit Bonds Payable

or

> Debit Cash
> > Credit Bond Premium
> > Credit Bonds Payable

AMORTIZATION METHODS

The bond discount or premium is amortized over the life of the bond. Accounting Principles Board Opinion Number 21 stated that this amortization should be determined "in such a way as to result in a constant rate of interest when applied to the amount outstanding at the beginning of any given period" (APB, 1971). The interest or effective interest method satisfies this requirement. Other methods, such as the straight-line method, "may be used if the results obtained are not materially different from those which would result from the 'interest' method" (APB, 1971).

The journal entries for recording bond interest expense through the use of either the straight-line or interest method are:

> Debit Bond Interest Expense
> > Credit Bond Discount
> > Credit Cash or Interest Payable

or

> Debit Bond Interest Expense
> Debit Bond Premium
> > Credit Cash or Interest Payable

The straight-line method is computationally easier to use than the interest method and is frequently used by educators to illustrate the concept of discount or premium

amortization. Accountants using this approach divide the initial unamortized bond discount or premium by the number of bond interest payments until the maturity date to determine the amortization amount for each payment period. This method results in an equal interest expense and amortization amount over the life of the bond.

Although the straight-line method is "user-friendly," the constant interest expense that is computed by the method is not consistent with the changing carrying value of the bond. The carrying value, which is equal to the bond principal less the unamortized bond discount or plus the unamortized premium, changes each interest payment and amortization period. Accordingly, as the carrying value changes, the interest rate associated with the bond must vary to keep the interest expense constant. This result does not conform to the financial accounting requirement of maintaining a constant interest rate for a bond issue.

THE INDIRECT INTEREST METHOD OF AMORTIZATION

The interest method for computing interest expense addresses this problem. Interest expense is calculated by multiplying the carrying value for an interest payment period by the constant market interest rate, resulting in a varying interest expense each payment period. Application of this method meets the requirement for a constant interest rate to be used over a bond's life. It is common for accounting textbooks and computerized spreadsheets to describe discount and premium amortizations using the interest method as the difference between the computed interest expense and the interest paid or accrued. For partial amortizations between interest payment dates, Budak et al. determined that the straight-line method may be used (Budak, 2002).

Usage of the interest method is consistent with APB Opinion Number 12, which states that "the objective of the interest method is to arrive at a periodic interest cost (including amortization) which will represent a level effective rate on the sum of the face amount of the debt and (plus or minus) the unamortized premium or discount and expense at the beginning of each period. The difference between the periodic interest cost so calculated and the nominal interest on the outstanding amount of the debt is the amount of periodic amortization" (APB, 1968). Amortization schedules indicate that the user must simply subtract the values in the interest expense column from the values in the payment column to compute the amortization amount.

The assumption inherent in this approach is that the expense and payment amounts have been correctly calculated, since amortization is a residual of the process. An alternative method, however, provides a direct calculation that is independent of the amounts computed for interest expense and payment. The user of this method, whether a student or practitioner, is able to avoid the problem of computing amortizations incorrectly when the value determined for either the expense or payment is erroneous.

THE DIRECT INTEREST METHOD OF AMORTIZATION

This alternative method is based on the fact that the value of a bond is equal to the sum of two present values: the present value of the single principal payment at maturity and the present value of the annuity representing the constant interest payments over the life of the bond. Present value factors are determined by using the market interest rate on the bond issuance date and the number of bond interest payments. The market interest rate is converted from an annual rate to the rate that is appropriate for the interest payment periods by, for example, dividing the annual rate by two for semiannual interest payment periods.

The derivation of the formula for the direct calculation of discount and premium amortizations using the interest method begins with the recognition that the amount amortized is the difference between the present values of the bond at the end of the interest payment period and at the beginning of the period. The formulas for the present value of a single payment and the present value of an ordinary annuity can be used to calculate this difference. The steps of the derivation are presented in appendix 1.

The mathematical development in the appendix could be used by accounting educators as an example of the application of elementary algebraic methods in an accounting course. Since the topic of bonds is included in accounting texts in the context of time value of money, students would gain an additional opportunity to strengthen their understanding of this subject. Also, accounting instructors usually introduce the topic of discount and premium amortizations by discussing the direct calculation of amortization amounts using the straight-line formula. Instructors would be able to transition to a similar formula-based approach for computing interest-method amortizations directly and not require students to follow the steps of the indirect method.

The direct-method formula is not limited to these educational applications. It provides users with the option to verify independently the results computed by the indirect method. The amortization for any interest payment period selected during the life of the bond is readily obtainable by direct substitution in the formula. The accountant using this method has the flexibility of computing any interest payment period's interest expense by combining the amortization amount with the interest paid or accrued. Also, the results from the formula allow comparisons to be made between the straight-line and interest methods to determine if the results are materially different.

FORMULA FOR DIRECT COMPUTATION OF AMORTIZATIONS USING THE INTEREST METHOD

The formula for computing discount and premium amortizations on semiannual interest payment dates requires the user to know the market and coupon interest rates, the bond

principal, and the single-payment present value factor. It is not necessary to know either the interest expense or payment. The variables used in the formula are: N = Total number of interest payments over the life of the bond, n = Period for the amortization calculation, i_m = Market or effective annual rate of interest expressed as a decimal, i_c = Coupon annual rate of interest expressed as a decimal, P = Principal or face amount of the bond, and PV $_{N-n+1}$ = Present value factor for a single payment of $1.00 for the period N-n+1 using the market interest rate. The formulas for computing semiannual discount or premium amortizations directly using the interest method are:

Amortization for any Period n = $(PV_{N-n+1})(P)(i_c - i_m)/2$ (1)

The formula can be restated as:

Amortization for any Period n = $P(i_c - i_m) \div 2(1 + i_m/2)^{N-n+1}$ (2)

If $i_c > i_m$, $i_c - i_m > 0$ and the positive amortization amount would indicate that the bond was issued at a premium. If $i_c < i_m$, $i_c - i_m < 0$, and the negative amount would indicate that the bond was issued at a discount. If $i_c = i_m$, $i_c - i_m = 0$ and the amortization amount is zero, indicating that the bond was issued at face value.

COMPUTATIONAL EXAMPLE

For example, if interest is paid semiannually on a $100,000, 20-year bond issued on the interest payment date with a five percent coupon rate when the market interest rate was six percent, the first interest payment period discount amortization would be:

Amortization = $(PV_{40})(100,000)(0.05 - 0.06)/2 = -(0.30656)(100,000)(0.005) = -153.28$.

(The negative sign indicates that the value is for discount amortization.)

Using the indirect method, the user must first compute the carrying value:

Carrying Value = PV (Principal) + PV (Annuity for Interest Payments) =

$(0.30656)(100,000) + (23.11477)(100,000)(0.025) = 30,656 + 57,786.93 = 88,442.93$.

Interest Expense = $(88,442.93)(0.03) = 2,653.29$, and interest paid = 2,500.

Therefore, amortization = 2,653.29 - 2,500 = 153.29. Since the interest paid is less than the interest expense, the user of this formula concludes that the bond was issued at a discount.

Alternatively, using Formula (2),

Amortization = $(100,000)(0.05 - 0.06) \div [2(1 + 0.03)^{40}]$ = $-(1,000) \div 2(3.26204)$
= -153.28.

The second interest payment period discount amortization using the direct method would be:

Amortization = $(PV_{39})(100,000)(0.05 - 0.06)/2$ = $-(0.31575)(100,000)(0.005)$ = -157.88.

Using the indirect method, the new carrying value is the sum of the first-period carrying value and the discount amortization:

Carrying Value = 88,442.93 + 153.29 = 88,596.22.

Interest Expense = $(88,596.22)(0.03)$ = 2,657.89, and interest paid = 2,500. Amortization for this second-period interest payment = 2,657.89 - 2,500 = 157.89.

Using Formula (2),

Amortization = $(100,000)(0.05 - 0.06) \div [2(1 + 0.03)^{39}]$ = $-(1,000) \div 2(3.16703)$ = -157.88.

Most student calculators have a "y^x" key that allows direct computation of the higher-value exponents associated with Formula (2). Also, users of this formula could enter it in spreadsheet programs to generate amortization tables for various input values.

Appendix 2 illustrates the direct and indirect amortization results for two bond issues and compares the formula-computed values to the values computed indirectly by subtracting the interest payment and expense amounts.

CONCLUSION

The test results comparing the direct amortization method to the indirect method confirm that the two methods yield the same answers. The direct method provides a one-step computational approach, which allows the user to avoid the multiple steps and potential errors associated with the indirect method. Instructors and authors can refer to the mathematical development of the indirect method formula to illustrate the quantitative relationships that support the calculation of discount and premium amortizations through the use of the interest method. The flexibility of the formula allows users to compute amortizations for any interest-payment period during a bond's life without deriving carrying values initially. This capability supports account analysis as well as financial planning associated with debt issuances.

DERIVATION OF THE AMORTIZATION FORMULA

The carrying value of a bond at any time during its life is equal to the sum of the present values of the interest payments remaining until the maturity date of the bond and the present value of the principal payment at maturity. The interest payments over the bond's life can be represented as an ordinary annuity, and the present value of the sum of the payments of \$1 each period for n periods at a market interest rate i_m is

$$\frac{1 - (1 + i_m)^{-n}}{i_m} \quad \text{or} \quad i_m^{-1}(1 - (1 + i_m)^{-n}).$$

For an interest payment of C dollars per period, this formula becomes

$i_m^{-1}(1 - (1 + i_m)^{-n})\, C$. Using the exponential form, the present value (PV) of a single payment of \$1 at the end of period n and at an interest rate of i_m is

$(1 + i_m)^{-n}$. For a principal payment of P dollars on the maturity date, this becomes

$(1 + i_m)^{-n}\, P$.

The carrying value for any period n of a bond paying C dollars in interest per period and P dollars on the maturity date, therefore, is

$$i_m^{-1}(1 - (1 + i_m)^{-n})\, C + (1 + i_m)^{-n}\, P.$$

The amortization of bond discounts or premiums equals the difference between the carrying value at the end of the amortization period and the carrying value at the beginning of the period. If A_n is the amortization amount for any period n,

$$A_n = [i_m^{-1}(1 - (1 + i_m)^{-n})\, C + (1 + i_m)^{-n}\, P] - [i_m^{-1}(1 - (1 + i_m)^{-n+1})\, C + (1 + i_m)^{-n+1}\, P]$$

$$= i_m^{-1}\, C[(1 - (1 + i_m)^{-n}) - (1 - (1 + i_m)^{-n+1})] + P[(1 + i_m)^{-n} - (1 + i_m)^{-n+1}]$$

$$= i_m^{-1}\, C\, (1 + i_m)^{-n}[-1 + (1 + i_m)] + P\, (1 + i_m)^{-n}[(1 - (1 + i_m))]$$

$$= i_m^{-1}\, C\, (1 + i_m)^{-n}(i_m) - P\, (1 + i_m)^{-n}(i_m)$$

$$= C\, (1 + i_m)^{-n} - i_m\, P\, (1 + i_m)^{-n}$$

$$= (1 + i_m)^{-n}(C - i_m\, P)$$

$$= PV\, (C - i_m P).$$

<u>Appendix 1 continued</u>

If the face or coupon interest rate is i_c, the interest payment is $C = i_c\, P$.

Therefore, $A_n = (PV)(i_c\, P - i_m\, P) = (PV)(P)(i_c - i_m)$.

Since the bond carrying value depends on the present value of interest and principal cash flows after an interest payment period, the present value factor PV should reflect this future time horizon. The present value for the period N-n+1, where N is the total number of interest payments over the life of the bond, is the factor for the periods remaining after the first n periods. If interest is paid semiannually, formula (1) is

$A_n = (PV_{N-n+1})(P)(i_c - i_m)/2$.

Alternatively, formula (2) may be derived by substituting for PV_{N-n+1},

$A_n = P(i_c - i_m) \div 2(1 + i_m/2)^{N-n+1}$.

<u>Appendix 2</u>

The premium amortization amounts for several interest payment dates for a 20-year, $10,000 bond with a six percent coupon rate are presented in table 1. The market interest rate on the bond issuance date was five percent, and interest is paid semiannually (N = 40).

<p align="center">Table 1</p>

Interest Payment Period (n)	Direct Method (Formula 2)	Premium Amortization Amount ($)			
		Indirect Method			
		Carrying Value	Interest Expense	Interest Paid	Amortization
1	18.62	11,255.14	281.38	300	18.62
5	20.55	11,177.82	279.45	300	20.55
10	23.26	11,069.79	276.74	300	23.26
20	29.77	10,809.26	270.23	300	29.77
25	33.68	10,652.78	266.32	300	33.68
40	48.78	10,048.78	251.22	300	48.78

Table 2 presents the discount amortization results for several interest payment dates for a 10-year, $5,000 bond with a two percent coupon rate. The market interest rate on the issuance date was four percent, and interest is paid semiannually (N = 20).

Table 2

		Discount Amortization Amount ($)			
Interest	Direct		Indirect Method		
Payment	Method	Carrying	Interest	Interest	
Period (n)	(Formula 2)	Value	Expense	Paid	Amortization
1	33.65	4,182.43	83.65	50	33.65
5	36.42	4,321.11	86.42	50	36.42
10	40.21	4,510.64	90.21	50	40.21
20	49.02	4,950.98	99.02	50	49.02

Table 3 illustrates the application of present value factors to the data for table 1.

Table 3

Interest Payment Period (n)	Present Value Factor for 41 − n, 2.5%	$P(i_c - i_m)/2$	Amortization ($)
1	0.37243	50	18.62
5	0.41109	50	20.55
10	0.46512	50	23.26
20	0.59539	50	29.77
25	0.67363	50	33.68
40	0.97561	50	48.78

Table 4 illustrates the application of present value factors to the data for table 2.

Table 4

Interest Payment Period (n)	Present Value Factor for 21 − n, 2%	P(i$_c$ − i$_m$)/2	Amortization ($)
1	0.67297	50	33.65
5	0.72845	50	36.42
10	0.80426	50	40.21
20	0.98039	50	49.02

REFERENCES

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3. Accounting Principles Board, *Opinion No. 12-Omnibus Opinion-1967*. New York: American Institute of Certified Public Accountants, Inc. (1968), par. 16.